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MINIMUM STEPS REQUIRED WHEN ESTIMATING GPS-DERIVED ORTHOMETRIC HEIGHTS

David B. Zilkoski
NAVD 88 Project Manager
Vertical Network Branch
National Geodetic Survey
Rockville, Maryland 20852

BIOGRAPHICAL SKETCH

David B. Zilkoski received a B.S. degree in Forest Engineering from the College of Environmental Science and Forestry, Syracuse, New York, in 1974, and an M.S. degree in Geodetic Science from the Ohio State University in 1979. He has been employed by the National Geodetic Survey (NGS) since 1974. From 1974 to 1981, as a member of the Horizontal Network Branch, he participated in the new adjustment of the North American Datum of 1983. In 1981, he transferred to the Vertical Network Branch and served as Chief, Vertical Analysis Section, until 1986. His present position is Geodesist, Vertical Network Branch, and Project Manager, New Adjustment of the North American Vertical Datum.

Mr. Zilkoski is a member of the American Congress on Surveying and Mapping (ACSM) and is an instructor for the NGS Vertical Control Workshop and ACSM-NGS Surveying Instrumentation and Coordinate Computation Workshops. He is also a member of the American Geophysical Union and President of International Association of Geodesy Special Study Group 1.102, "Vertical Reference Systems."

ABSTRACT

It is clear that GPS-derived orthometric heights will have a major impact on the surveying community in the near future. Results of analyses performed by NGS indicate that with the use of appropriate planning and proper strategy when estimating geoid height differences, it is possible to compute GPS-derived orthometric heights that meet a wide range of engineering and land surveying requirements for vertical control. NGS is currently working on algorithms and models to improve the estimation of geoid heights and geoid height differences. The minimum steps required when estimating GPS-derived orthometric heights are discussed.

INTRODUCTION

Analysis of Global Positioning System (GPS) survey data has shown that GPS can be used to establish precise relative positions in a three-dimensional Earth-centered coordinate system. GPS carrier phase measurements are used to determine vector base lines in space, where the components of the base line are expressed in terms of cartesian coordinate differences. These vector base lines can be converted to differential latitude and longitude, and ellipsoidal height difference (dh) relative to a defined reference ellipsoid.

An orthometric height difference (dH) can then be obtained from an ellipsoid height difference by subtracting the geoid height difference (dN):

$$dH = dh - dN.$$

(Note: this is an approximate equation but the error is always very small and is insignificant for practical applications.)

It is clear that GPS-derived orthometric heights will have a major impact on the surveying community in the future. The minimum steps required to compute useful GPS-derived orthometric heights are discussed. The list is not meant to be complete, it is only a minimum set of steps which need to be performed for all projects.

HEIGHTS AND HEIGHT DIFFERENCES

Orthometric heights (H) are referenced to an equipotential surface, e.g., the geoid, which approximates the mean sea level. The orthometric height of a point on the Earth's surface is the distance from the geoid to the point, measured along the plumb line normal to the geoid. Ellipsoid heights (h) are referenced to a reference ellipsoid. The ellipsoid height of a point is the distance from the reference ellipsoid to the point, measured along the line which is normal to the ellipsoid. At a point, the difference between the ellipsoid height and the orthometric height is defined as the geoid height (N) (to a sufficient approximation).

When high-accuracy field procedures are used, orthometric height differences can be computed from measurements of precise geodetic leveling with an uncertainty of less than 1 cm over a 50-kilometer distance. Less accurate results are achieved when third-order leveling methods are employed. Depending on the accuracy requirements, GPS surveys and present geoid prediction models can be employed as an alternative to classical geodetic leveling methods, but are not as accurate. The current limiting factor is the accuracy of estimated geoid height differences. The models used to estimate geoid heights in the United States are too generalized to accurately represent the local relief of the geoid. In most regions of the United States, over very small areas, i.e., 10-km by 10-km, the relief of the geoid can usually be assumed to be flat. It has been shown that when proper GPS field procedures are followed and a significant number of vertical control points are occupied by GPS (thereby allowing geoid heights to be interpolated with sufficient accuracy), it is possible to compute GPS-derived orthometric heights which will meet a wide range of existing engineering projects' vertical control requirements (Zilkoski and Hothem 1989, Zilkoski 1990a, and Hajela 1990).

STEPS REQUIRED WHEN ESTIMATING GPS-DERIVED ORTHOMETRIC HEIGHTS

An important aspect of any geodetic positioning technique is to ensure that all data outliers have been removed from the data. GPS results can be evaluated by analyzing network loop misclosures, repeat base line differences, and least squares adjustment results. The design of the network should be such that there are enough redundant observations to detect data outliers.

The <u>minimum</u> steps required when analyzing GPS-derived orthometric heights are listed below.

1. During the project's <u>planning</u> stage, perform a detailed analysis of the geoid in the area of the survey in order to determine if

additional gravity and/or leveling data are required to adequately estimate the slope of the geoid and changes in slope.

a. Perform a detailed study of the density and distribution of observed gravity values and plot free-air anomalies to determine where changes in slope of the geoid may exist.

2. During the project's planning stage, perform a detailed study of the leveling network in the area, i.e., plot all leveling lines, note the age of leveling data, determine if bench marks can be occupied by GPS receivers, etc.

a. Perform a history check on monuments to determine if they are stable bench marks and if they are referenced to the same vertical datum.

3. Perform a 3-D minimum constraint least squares adjustment of the GPS data.

a. Compare GPS-derived coordinates with results of higher-order surveys to determine if coordinates (latitude, longitude, and elliposid height) estimated from higher-order surveys can be used to control errors in lower-order surveys.

4. Using the best available geoid heights, compare adjusted GPS-derived orthometric height differences obtained from step 3 with leveling-derived orthometric height differences.

5. Detect and remove all data outliers determined in steps 3 and 4.

6. Analyze the local geoid in detail.

a. Plot the geoid heights in the area (see figure 1).

b. Plot the estimated slope of the geoid using differences between GPS-derived ellipsoid height differences and leveling-derived orthometric height differences ($dN = dh - dH$) obtained in step 4.

7. Estimate GPS-derived orthometric heights and local systematic errors in the geoid heights by solving for the geoidal slope and scale using the method described by Vincenty (1987) and demonstrated in Zilkoski and Hothem (1989) and Zilkoski (1990a).

8. Compare adjusted GPS-derived orthometric height differences from step 7 with leveling-derived orthometric height differences to determine appropriate scale and rotation parameters.

a. Determine if there is enough valid information to reliably estimate scale and rotation parameters. Large differences between GPS-derived orthometric heights and leveling-derived orthometric heights may be due to "bad" information. Bad information includes, but is not limited to, the following: bench marks which have moved since their heights were last determined, misidentified stations, inconsistent vertical datums, incorrect ellipsoid heights, and low-resolution geoid models.

b. Using the results of steps 6 through 8, determine if the slope of the geoid changed within the project's boundaries, in order to solve for additional parameters.

9. Compute GPS-derived orthometic heights by performing a 3-dimensional least squares adjustment holding fixed all appropriate orthometric height values of published bench marks (and appropriate GPS-derived coordinates (latitude, longitude, and ellipsoid height) computed from higher-order surveys) and solving for appropriate scale and rotation parameters.

10. Use the results from steps 1 through 9 to document the estimated accuracy of the GPS-derived orthometric heights.

Of course, it must be understood that each project is different and, therefore, the procedures used to compute GPS-derived orthometric heights will be slightly different for each project. There is not a simple "cookbook" method that works well all the time everywhere. The results of all steps and comparisons with known values must be considered before determining final GPS-derived heights. NGS and others are working on algorithms and models to improve the computation of geoid heights and geoid height differences (Nagy and Fury 1990, Milbert 1990). NGS is actively pursuing an integrated geodesy approach of combining leveling data and GPS measurements with gravity data to solve for an improved geoid (Milbert and Dewhurst 1990).

DISCUSSION OF MINIMUM STEPS

This report discusses the first five steps in more detail. The remaining steps are described and documented in other reports (Zilkoski and Hothem 1989, Zilkoski 1990a, and Hajela 1990).

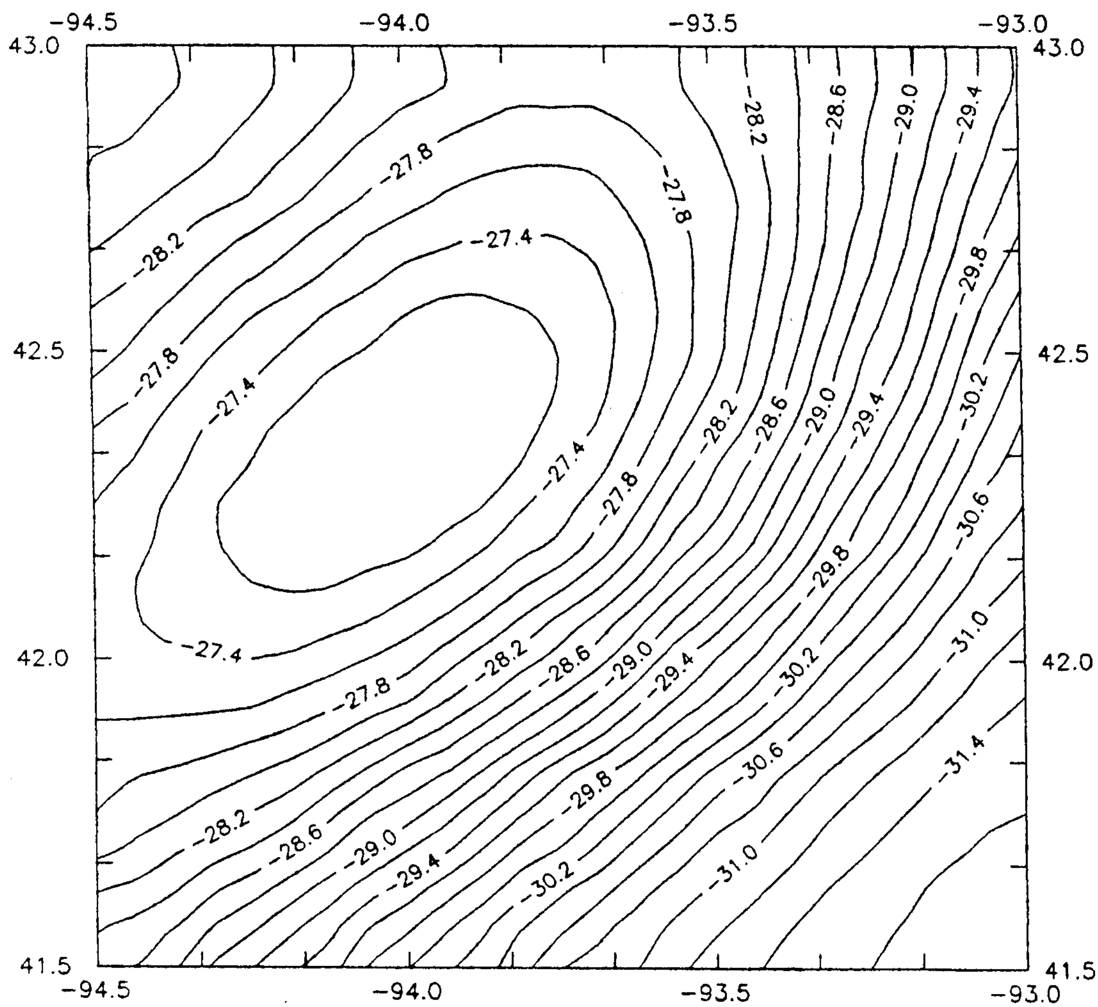

Figure 1. Plot depicting geoid heights estimated using the OSU89B (Rapp and Pavlis 1990) global geopotential model between latitudes 41.5°N and 43.0°N and longitudes 93.0°W and 94.5°W, units = meters.

Step 1, performing a detailed analysis of the geoid in the area of the survey, is an important step in determining GPS-derived orthometric heights. It is critical to determine which bench marks need to be occupied with GPS to adequately evaluate the shape of the geoid, i.e., its slope and changes in slope.

Geoid heights estimated using global geopotential models, e.g., OSU89B (Rapp and Pavlis 1990), may be too generalized to adequately model the local relief of the geoid. Plotting free air anomalies and gravity values in the area of the survey will help in determining where GPS stations with known (leveled) orthometric heights should be located. Figure 1 depicts the shape of the geoid in an area in Iowa using the OSU89B global geopotential model. Notice the fairly consistent slope of the geoid on the right side of the plot and the prominent changes in slope on the upper left half of the plot.

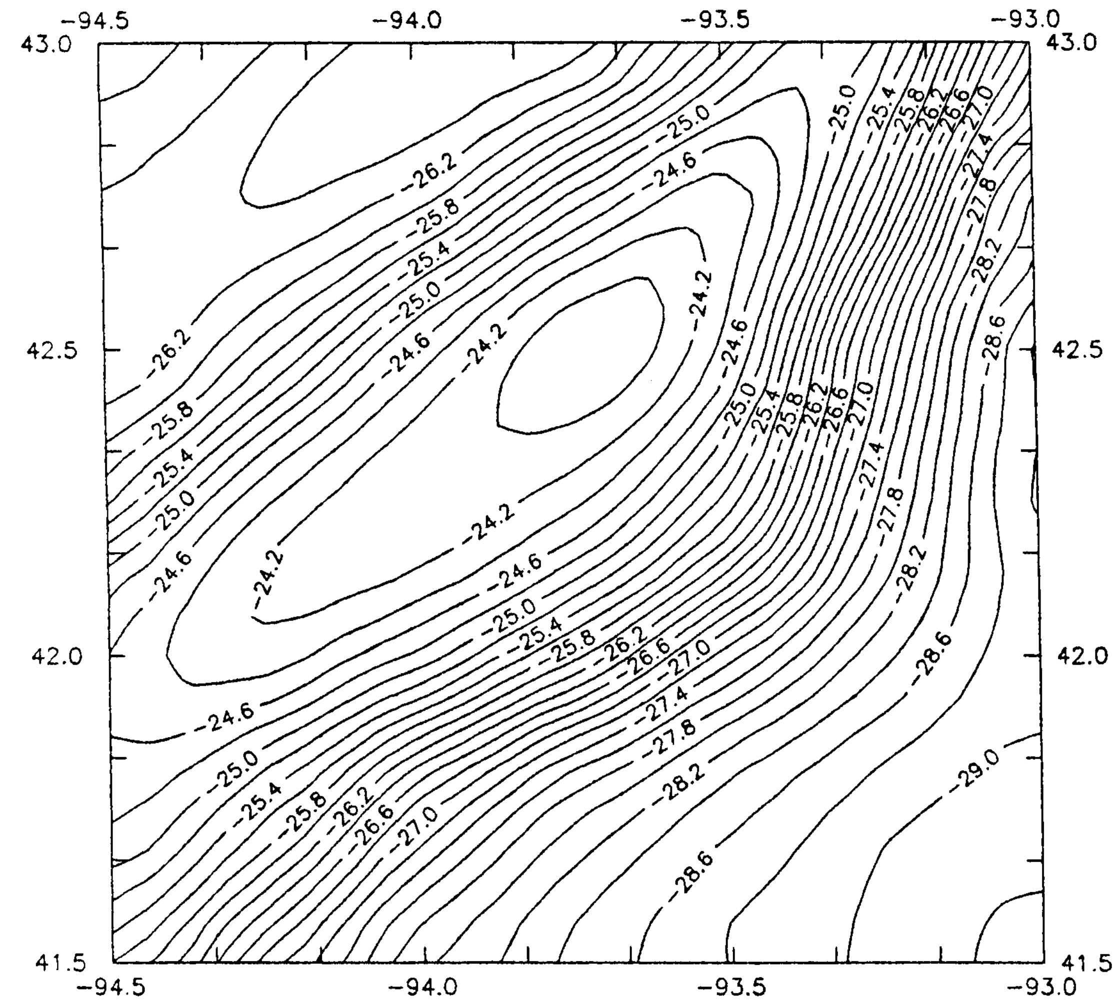

Figure 2. Contour plot depicting geoid heights estimated using the Fast Fourier Transform (FFT) technique between latitudes 41.5°N amd 43.0°N and longitudes 93.0°W and 94.5°W, units = meters.

Figure 2 is a contour plot of geoid heights for the same area shown in figure 1 computed from gravity using the Fast Fourier Transform (FFT) technique (Nagy and Fury 1990). Figure 3 is a plot of free-air gravity anomalies of the same area. Notice the differences in relative geoid heights depicted in figures 1 and 2. The FFT solution provides a much better determination of the geoid heights than the global geopotential model solution. The plot of free air anomalies indicates that the shape of the geoid is complex in this area and stations with known orthometric height values should be occupied with GPS wherever the contours indicate a change in slope. Notice that the contour plots given in figures 2 and 3 have very similar shapes.

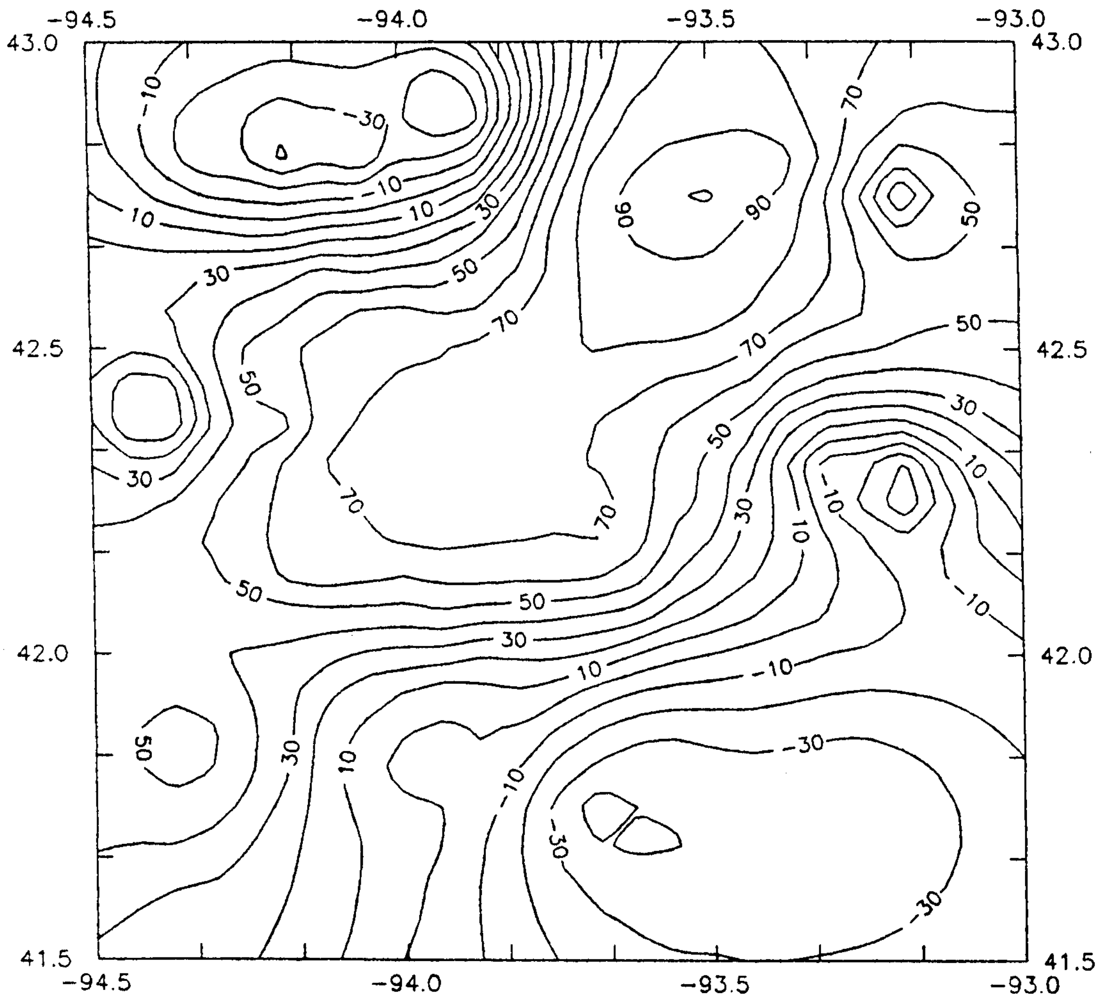

Figure 3. Contour plot depicting free-air gravity anomalies between latitudes 41.5°N and 43.0°N and longitudes 93.0°W and 94.5°W, units = mgals.

Figure 4a is a cross-sectional plot of figures 1 and 2 at latitude 42.5 degrees. Notice that the long wavelength feature of the geoid was recovered in both the OSU89B and FFT solutions, but that the local differences exceed 1 meter. NGS is currently working on producing a high-resolution geoid height model for the continental United States (Milbert 1990). Even after this geoid model is routinely disseminated by NGS, users will still need to occupy bench marks strategically located in the area of the survey to verify the slope and changes in slope of the modeled geoid.

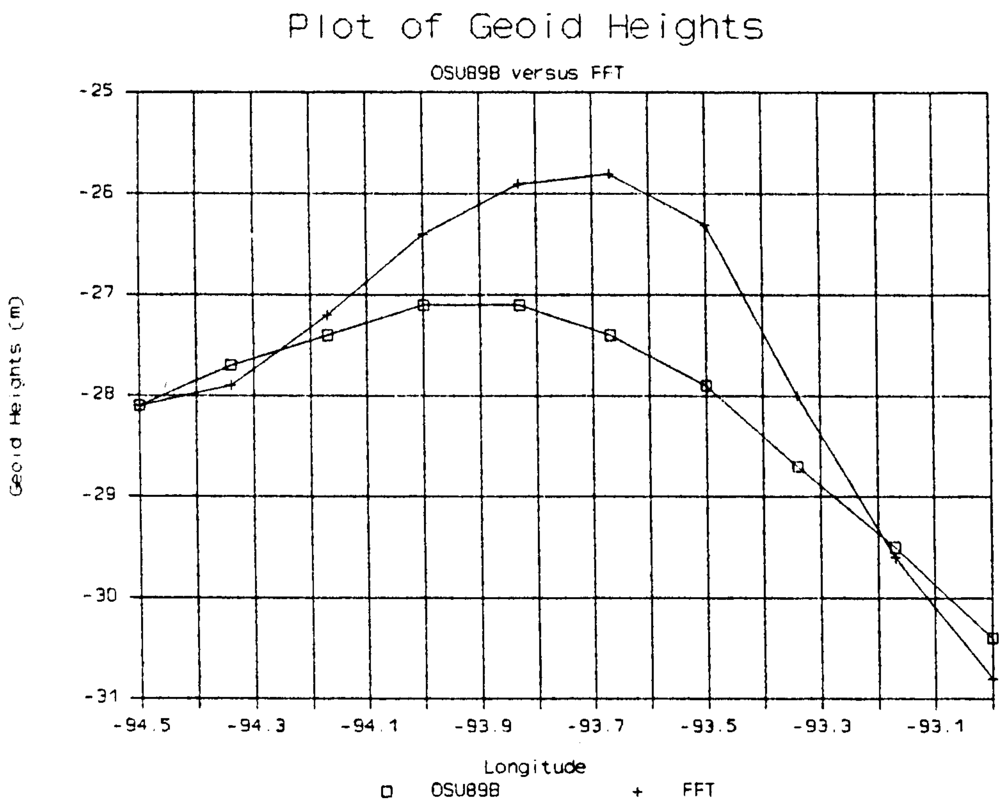

Figure 4a. Cross-sectional plot comparing geoid height values estimated using the OSU89B model and FFT technique at latitude 42.5°N between longitudes 93.0°W and 94.5°W.

Figure 4b is a plot that compares geoid height estimates using the OSU89B model, FFT technique, and GPS-derived ellipsoid heights minus published orthometric leveled heights (h - H) for five bench marks in a GPS project in Iowa. The latitudes of the stations range between 42° 28' and 42° 41'. The ellipsoid heights were estimated from a 3-D minimum-constraint least squares adjustment of the GPS data (Prusky 1990) and the orthometric heights of the bench marks were published NGVD 29 values. If it is assumed that the (h - H) values are the best estimate of geoid heights for these five bench marks, then the FFT technique provides a better estimate of local geoid heights in this area than the OSU89B model. This was expected and agrees with the results shown in figure 4a. Notice that the FFT solution does a good job of representing the slope and changes in slope of the geoid. This is important when estimating scale and rotation parameters because an incorrect assumption about the change in slope of the geoid can cause significantly large errors in GPS-derived orthometric heights (Zilkoski 1990a).

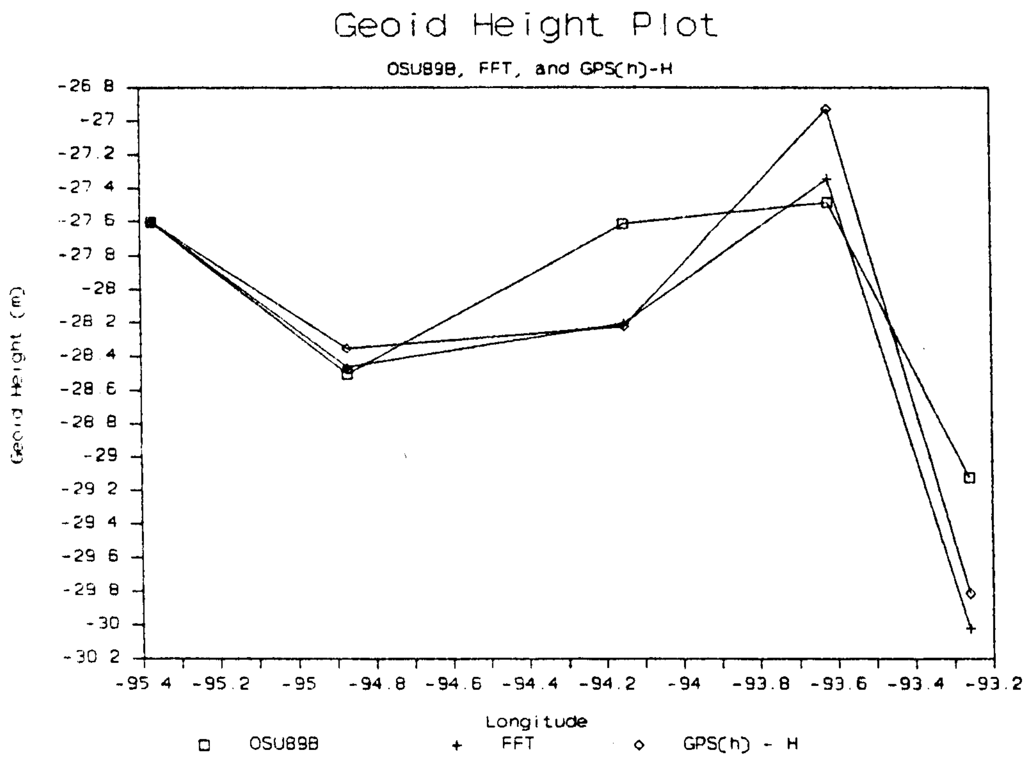

Figure 4b. Plot comparing geoid height values estimated using the OSU89B and FFT technique at five bench marks in IOWA.

Bench mark movement is an error source that many analysts ignore, or do not have enough information to evaluate properly. The profile in figure 5 depicts the differences in heights of bench marks determined by two different epochs of leveling data. It is obvious from figure 5 that bench mark R 5 was physically disturbed between epochs 1957 and 1975. The latest estimate of the height for R 5 differs from the previous value by approximately 20 cm. If the incorrect older height were held fixed, the 20 cm would be forced into the adjustment of GPS data and would distort other GPS-derived ellipsoid and orthometric heights in the network. Another possible problem, and one which cannot be determined from the available data, is whether R 5 has moved since epoch 1975.

The recommended procedure to check for movement of bench marks is to perform check leveling between two or more bench marks and compare the results with published values. Another method that can be used, which may be less expensive during a GPS survey, is to occupy two bench marks that are only 2-3 km apart using GPS. With several new efficient GPS techniques becoming operational, e.g., kinematic and pseudo-static GPS (Remondi 1988), these additional "GPS-leveling" ties should not require an excessive amount of additional resources. The geoid height differences over small areas in most regions of the United States should be small enough that ellipsoid height differences can be compared with published orthometric height differences to check the stability of bench marks to the 5-to-10 centimeter level.

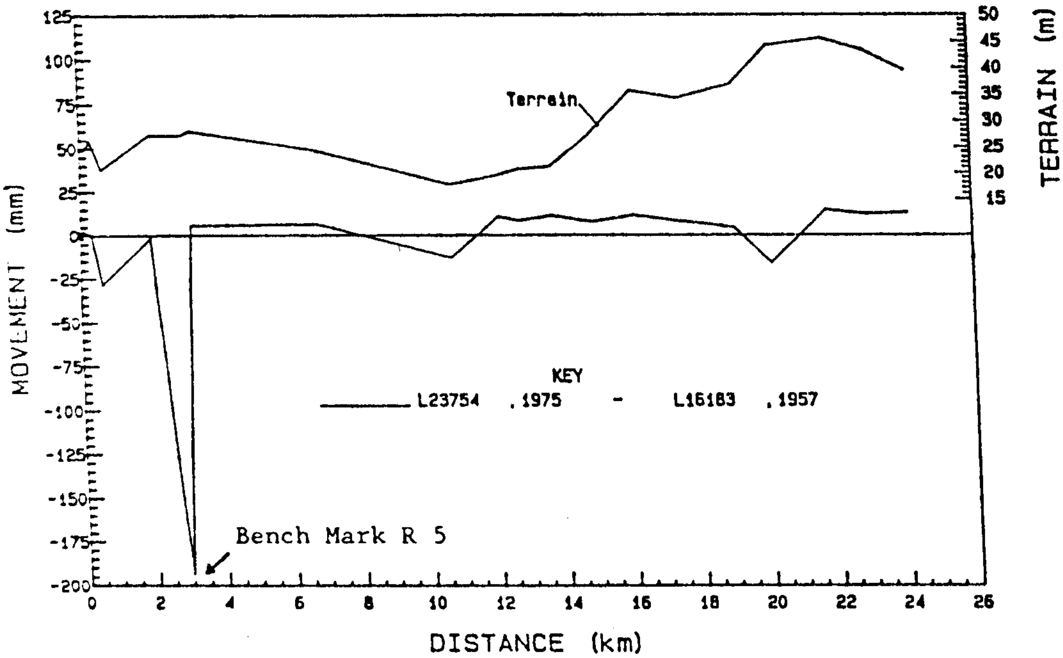

Figure 5.--Profile depicting differences in heights of bench marks computed from two different epochs (1957 and 1975) of leveling data near the GA/FL state line in the Folkston, GA, area.

The analyst must also ensure that all bench mark values are referenced to the same vertical reference system, e.g., NGVD 29, and that published values do not contain inconsistencies due to previous adjustment constraints. This is not a major problem for bench marks published by NGS, but there are a few inconsistencies in NGVD 29. This is one of the reasons NGS is performing the new adjustment of the North American Vertical Datum of 1988 (Zilkoski and Young 1985, and Zilkoski 1990b). An example of one inconsistency in NGVD 29 is near Oak Hill, Florida (Zilkoski 1990a). Here, between bench marks D 227 and JLR 370, which are only 0.85 km apart, a 10-centimeter difference exists between published NGVD 29 height differences and adjusted height differences computed in a special minimally constrained test adjustment of the Florida primary leveling network. (See table 1.)

Table 1. An example of inconsistency in NGVD 29 (Florida).

Bench Mark	Special Adjusted Height (m)	Published NGVD 29 Height (m)	Difference Between Special Adj. and Published Height (cm)	Second Difference (cm)
D 227	3.624	3.624	0.0	
JLR 370	3.296	3.192	10.4	-10.4
J 211	3.502	3.405	9.7	0.7
JLR 371	3.358	3.263	9.5	0.2
HALE RM 2	2.882	2.884	-0.2	9.7

Similarly, a 10-centimeter difference also exists between stations JLR 371 and HALE RM 2. This inconsistency was introduced into NGVD 29 when

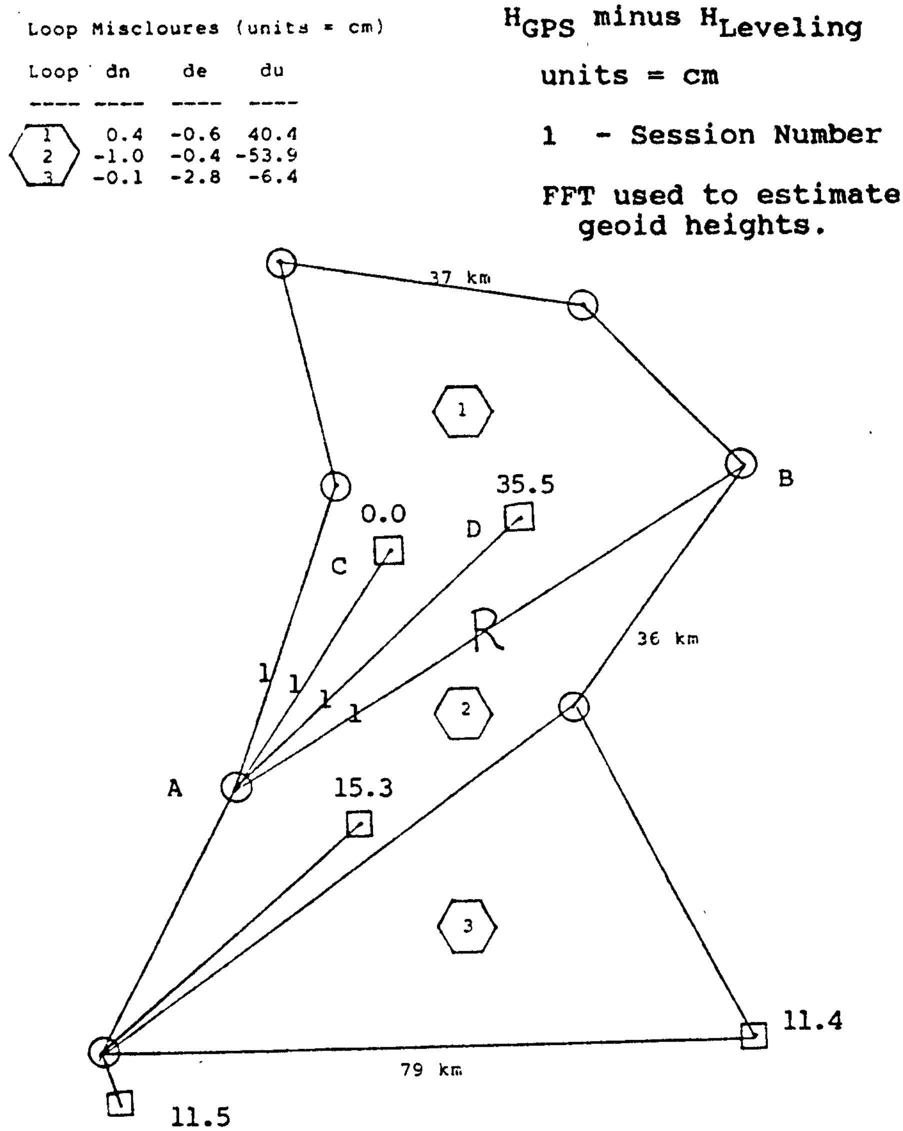

Figure 6. Sketch of GPS network comparing GPS-derived orthometric heights with leveling-derived orthometric heights and loop associated misclosures.

Bench mark movement is an error source that many analysts ignore, or do not have enough information to evaluate properly. The recommended procedure to check for movement of bench marks is to perform check leveling between two or more bench marks and compare the results with published values. Another method that can be used, which may be less expensive during a GPS survey, is to occupy two nearby bench marks (e.g., 2-3 km apart) using GPS. The geoid height differences over small areas in most regions of the United States should be small enough that ellipsoid height differences can be compared with published orthometric height differences to check the stability of bench marks to the 5-to-10 centimeter level. The analyst must also ensure that all bench mark values are referenced to the same vertical reference system, e.g., NGVD 29, and that published values do not contain inconsistencies due to previous adjustment constraints.

newer second-order area work was incorporated into older primary leveling data. Bench marks JLR 370, J 211, and JLR 371 were leveled to during both the primary and secondary leveling projects. New heights were computed for these three bench marks, but the bench marks involved with the older primary leveling data were not made consistent.

It is not known exactly how many large inconsistencies exist in the present NGVD 29 published height values. Some detected in recent years during analyses performed in support of special adjustments include a 13-centimeter inconsistency near Milton, Florida; a 14-centimeter inconsistency in the Hampton Roads, Virginia, area; a 5-centimeter inconsistency at Colonial Beach, Virginia; a 3-centimeter inconsistency at Cole Point, Virginia; and a 4-centimeter inconsistency at Indian Head, Maryland. NAVD 88 analysis is certain to uncover more of these inconsistencies.

Users are usually not aware of these inconsistencies because NGS has periodically readjusted large portions of the vertical network, distributing the inconsistencies over large areas. NGS does not have the resources to continue to maintain NGVD 29 as in the past. Eventually there would be a large number of areas in which surveyors would not be able to check their work using the present NGVD 29. NAVD 88 is specifically designed to remove the inconsistencies and distortions in the present NGVD 29.

It was mentioned previously that the design of the network should be such that there are enough redundant observations to detect data outliers. Figure 6 shows how loop closures can be used to detect a "bad" observation. The misclosures of loops 1, 40.4 cm, and 2, -53.9 cm, indicate that the common vector, vector A to B, may have a 40 to 50 centimeter error in the height component. Notice that the horizontal components, i.e., dn and de, are small. It cannot be assumed that the error in the vertical component is small just because the horizontal component closures are small. Based on loop closure analysis, the vector between stations A and B should be rejected.

It is important to note that the vectors between stations A and C and stations A and D are no-check GPS spurs. These two vectors are not a part of any loops and they were not reobserved. They were, however, observed during the same session as the vector AB. It is possible that vectors AC and AD were contaminated with the same error source that contaminated vector AB. But because they are no-check GPS spurs, it is not possible to use loop closures to check the results. However, comparing the GPS-derived orthometric heights with leveling-derived orthometric heights indicates that vector AD may contain a similar error. Although, it is possible that bench mark D has moved since its height was last determined by leveling. The point that needs to be made here is that the design of the network is such that the coordinates of stations C and D can not be checked using the GPS data. This is not an ideal condition and should be avoided at all times.

CONCLUSION

It is clear that GPS-derived orthometric heights will have a major impact on the surveying community in the future. The minimum steps required to compute useful GPS-derived orthometric heights were discussed. The list is not meant to be complete, but is the minimum set of steps which should be performed for all projects.

Performing a detailed analysis of the geoid in the area of the survey is an important step in computing GPS-derived orthometric heights. It is critical to determine which bench marks need to be occupied with GPS to adequately evaluate the shape of the geoid, i.e., its slope and changes in slope. NGS is currently working on computing a high-resolution geoid for the continental United States. Efforts to improve the accuracies of geoid height differences will depend on overall national accuracy needs for GPS-derived orthometric heights and on the cost of differential leveling versus GPS and gravity survey methods.

REFERENCES

Hajela, D. 1990, Obtaining Centimeter Precision Heights by GPS Observations Over Small Areas: GPS World, January/February, pp. 55-59.

Milbert, D. G. 1990, A High-Resolution Geoid for the Continental United States, Draft NGS Internal Document.

Milbert, D. G. and Dewhurst, W. T. 1990, The Yellowstone-Hebgen Lake Geoid Obtained Through the Integrated Geodesy Approach: Journal of Geophysical Research, (Submitted 1989).

Nagy, D. and Fury, R.J. 1990, Local Geoid Computation from Gravity Using the Fast Fourier Transform Technique: Bulletin Geodesique, (Accepted May 15, 1990).

Prusky, J. A. 1990, Iowa FAA GPS Survey Computation of Horizontal Control Report, NGS Internal Report.

Rapp, R. H. and Pavlis, N. K. 1990, OSU89A/B Potential Coefficient Models, Dept. of Geodetic Science and Surveying, The Ohio State University, Columbus, Ohio.

Remondi, B W. 1988, Kinematic and Pseudo-Kinematic GPS: Proceedings of the Satellite Division's International Technical Meeting, Colorado Springs, Colorado, September 19-23, 115-122.

Vincenty, T. 1987, On the Use of GPS Vectors in Densification Adjustment: Surveying and Mapping. Vol. 47, No. 2, 103-108.

Zilkoski, D. B. 1990a, Establishing Vertical Control Using GPS Satellite Surveys: Proceedings of the 19th International Federation of Surveying Congress (FIG), Commission 5, pp. 281-294.

Zilkoski, D.B. 1990b, NAVD 88 and NGS' Responsibility to the Surveying and Mapping Community: Proceedings of the GIS/LIS '90 Fall Convention, Anaheim, California, November 7-10.

Zilkoski, D. B. and Hothem, L. D. 1989, GPS Satellite Surveys and Vertical Control: Journal of Surveying Engineering, Vol. 115, No. 2, May, pp. 262-282.

Zilkoski, D. B. and Young, G. M. 1985, North American Vertical Datum (NAVD) Update: Proceedings of the U.S. Army Corps of Engineers 1985 Survey Conference, Jacksonville, Florida, February 4-8.